Unit No.10 SIMPLE HARMONIC MOTION AND WAVES

Vibratory Motion:

The to and fro motion of a body which repeat its motion again and again is called vibratory motion.

Simple Harmonic Motion (SHM):

It is a type of vibratory motion in which acceleration of the body is directly proportional to its displacement from mean position and it is always directed towards the mean position.

$$a \propto -x$$

Here negative sign shows that acceleration is directed towards the mean position.

Vibration:

One complete round trip of a vibrating body about its mean position is called one vibration.

Time Period:

The time taken by a vibrating body to complete one vibration is called time period. It is represented by **'T'** and its unit is **"Second".**

Frequency:

The number of vibrations of a vibrating body in one second is called frequency. It is represented by **'f'** and its unit is **"hertz".**

Frequency and time period are reciprocal of each other and their products is always equal to one.

$$T = \frac{1}{f}$$

OR

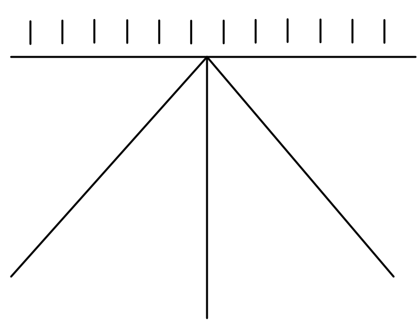
$$f = \frac{1}{T}$$

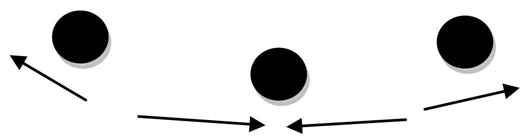
And

$$(T)(f) = 1$$

Amplitude:

The maximum displacement of a body from its mean position is called amplitude.





<u>Types of Waves:</u>

There are two main types of waves.

1. Mechanical Waves
2. Electromagnetic Waves

1. <u>Mechanical Waves</u>:

Such waves which require a certain medium for their propagation are called mechanical waves.

Examples:

Sound waves, water waves and waves produced on the strings are examples of mechanical waves.

2. <u>Electromagnetic Waves</u>:

Such waves which do not requires any certain medium for their propagation are called electromagnetic waves.

Examples:

Radio waves, television waves, x-rays, heat and light waves are examples of electromagnetic waves.

<u>Types of Mechanical Waves</u>:

There are two types of mechanical waves.

1. Transverse Waves
2. Longitudinal Waves

1. Transverse Waves:

Those mechanical waves in which particles of the medium vibrate perpendicular to the direction of proportional of waves are called transverse waves.

2. Longitudinal Waves:

Those mechanical waves in which particles of the medium vibrate parallel to the direction of proportional of waves are called transverse waves.

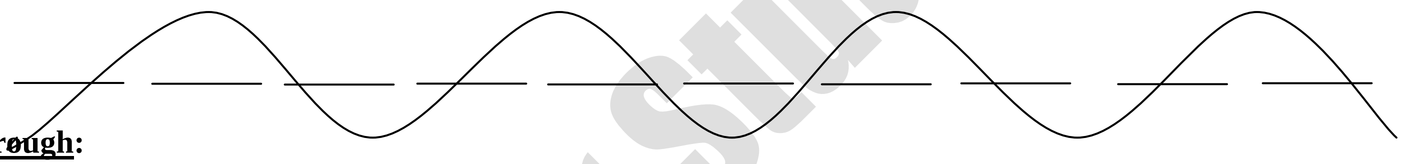
Q. Define following terms.

Ans. **<u>Crest</u>:**

Those parts of the transverse waves which are above the normal position are called crest. They are also called highest point of the medium.

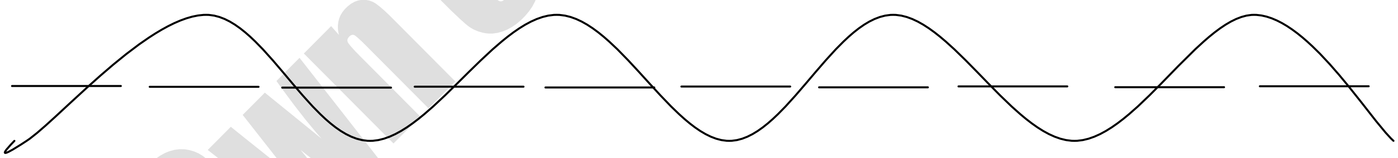
Crest **Crest** **Crest**

Crest



<u>Trough</u>:

Those parts of the transverse waves which are below the normal position are called trough. They are also called lowest point of the medium.



Trough **Trough** **Trough**

<u>Wavelength</u>:

The distance between two consecutive crests or two consecutive trough is called wavelength. It is represented by a Greek letter **"λ"**

<u>Question</u>:

Prove that $V = f\lambda$

OR

Write relation b/w wave speed wave frequency and wave length.

Ans. We know that the disturbance in a medium is called wave. When a wave travels from one place to another place it has a specific velocity. This is called velocity of wave and it is given by

$$Velocity = \frac{Distance}{Time}$$

If time taken by the wave in moving from one point to another point is equal to its time period 'T' then the distance covered by the wave will be equal to one wavelength 'λ' so in this case

$$Velocity = wave\ velocity = V$$
$$Distance = wavelength = \lambda$$
$$Time = Time\ period = T$$

Now eq (i) becomes

$$V = \frac{\lambda}{T}$$

We know

$$T = \frac{1}{f}$$

So

$$V = \frac{\lambda}{\frac{1}{f}}$$

$$V = f\left(\frac{\lambda}{1}\right)$$

$$V = f\lambda \qquad \text{Proved}$$

<u>Question</u>: What is S.H.M show that motion of mass attached with spring is S.H.M.

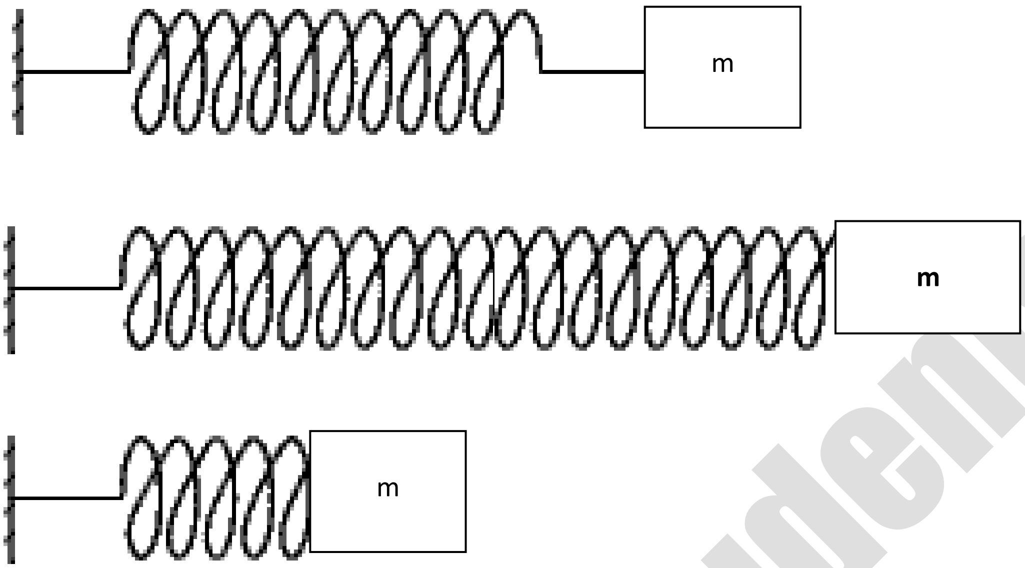
Ans. <u>Simple Harmonic Motion and Waves</u>

It is a type of vibratory motion in which acceleration of vibratory body is directly proportional to its displacement from mean position and it is always directed towards its mean position.

$$a \propto -x$$

Here – ive sign shows that acceleration is directed towards its mean position.

<u>Motion of Mass Attached to Spring:</u>



Consider a horizontal mass spring system in which a body of mass ‘m’ is attached with one end of the spring while the other end of the spring is a attached with strong sport. Initially no force acts on the mass therefore the mass is placed at position ‘O’ at rest. This position of the mass is called mean position.

External Force and Hooke’s Law:

If we apply some external force towards right then length of the spring increases by an amount ‘x’ and according to Hooke’s law.

$$F \propto x$$

$$F = (constant)x$$

$$constant = k$$

So

$$F = kx$$

Here ‘k’ is called spring constant.

Restoring Force:

After releasing the external force the mass will move towards equilibrium position 'O' but it does not stop at 'O' and continue to move up to point 'B'. This motion of the mass is due to restoring force. This restoring force is equal but opposite to external force.

$$Restoring\ force = -[External\ force]$$

$$F \quad = - \ [k\ x] \qquad (i)$$

From Newton Second Law of Motion

$$F = ma \qquad (ii)$$

$$From \quad (i)\ and\ (ii)$$

$$ma = -kx$$

$$a = -\frac{kx}{m}$$

$$a = -\left(\frac{k}{m}\right)x$$

$$Here\ \frac{k}{m} = constant$$

So

$$a = -(constant)x$$

$$a \propto -x$$

The above relation show that acceleration of the mass attached with spring is directly proportional to its displacement from mean position and it is always directed towards its mean position. Hence we can say that motion of mass attached with spring is S.H.M.

Time Period:

Time period of mass attached with spring is given by

$$T = 2\,\pi\,\sqrt{\frac{m}{k}}$$

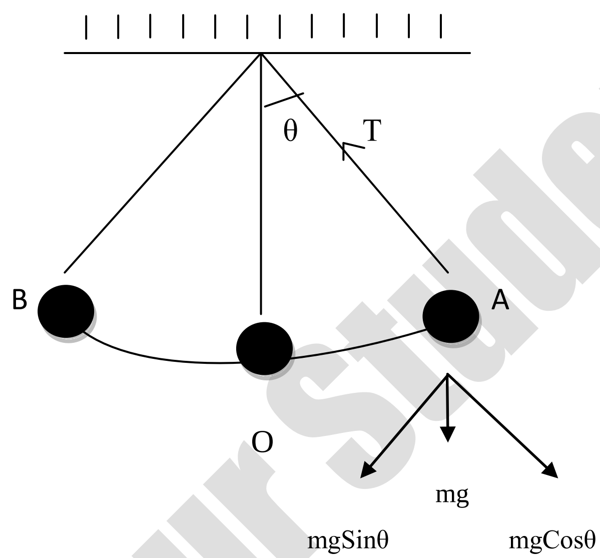
Question:

What is Simple Pendulum? Show that motion of simple pendulum is S.H.M.

Ans. Simple Pendulum:

Simple pendulum consists of a small bob of mass 'm' suspended by a light and inextensible string of length 'l' fixed at its upper end.

<u>Motion of Simple Pendulum</u>:



Consider a simple pendulum of length ‘l’ is held stationary in a vertical position ‘O’. This position of the bob is called mean position. If we apply some force ‘F’ and move the bob from its mean position ‘O’ to its extreme position ‘A’. After releasing the force the bob moves towards its mean position ‘O’ but it does not stop at ‘O’ and it will continue to move up to position ‘B’. Again from ‘B’ it will start moving towards ‘O’ and continue to move up to point ‘B’. In this way it will continue its motion b/w ‘A’ and ‘B’.

<u>Weight and Its Components</u>:

At point ‘A’ the weight of the bob is resolved into its rectangular components $mg\cos\theta$ and $mg\sin\theta$. The component $mg\cos\theta$ is along the string so it is cancelled by the tension of the string .The component $mg\sin\theta$ is directed towards mean position. Due to this component the bob will move towards its mean position.

<u>Maximum and Minimum Velocity</u>:

As the bob moves away from mean position its velocity decreases and as the bob moves towards its mean position its velocity increases. The velocity of the bob is maximum at mean position 'O' and it is minimum at its extreme position 'A' and 'B'.

Conclusion:

From above description it is clear that acceleration of the bob is directly proportional to its displacement from mean and it is always directed towards its mean position. Hence we can say that motion of simple pendulum is S.H.M.

Time Period:

Time period of simple pendulum is given by.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

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Damped Oscillations:

The oscillations of a system in the presence of some resistive force are called damped oscillations.

Damped Motion:

Such a motion of a body in which the friction reduces mechanical energy of the body with the passage of time is called damped motion. Due to damped motion the amplitude of vibrating body decreases.

Practical Applications of Damped Motion:

Shock absorbers in automobiles are one practical application of damped motion. A shock absorber of a car consists of a piston moving through a liquid such as oil. The upper part of the shock absorber is firmly attached to the body of the car. When the car travels over a bump on the road then the car may vibrate violently. But the shock absorber decreases the amplitude of these vibrations and converts mechanical energy of these vibrations into heat energy of the oil.

Waves As Carries of Energy:

Energy can be transferred from one place to another place through waves.

Therefore waves

are known as carries of energy.

Example:

When we shake the stretched string up and down then we provide some energy to the string. As a result of this energy a set of waves travels along the string. The vibrating force from the hand produces motion in the particles of the string. These particles then transfer their energy to the next particles. As a result energy transfers from one place to another place in the form of waves.

The amount of energy carried by the wave depends on the amplitude of the wave. If amplitude of the wave is greater than more energy can be transferred.

Important Information:

Christian Huygens invented the pendulum clock in 1956. He was inspired by the work of Galileo who had discovered that all pendulum of the same length took the same amount of time of complete one full swing Huygens developed the first clock that could accurately measure time.

Conceptual Questions:

10.1) If the length of the simple pendulum is doubled, what will be the change in its time period?

Ans. We know that Time period of simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If length is doubled then $l = 2l$

Now New time period is

$$T' = 2\pi\sqrt{\frac{2l}{g}}$$

$$T' = 2\pi\sqrt{2}\sqrt{\frac{l}{g}}$$

$$T' = \sqrt{2}\left[2\pi\sqrt{\frac{l}{g}}\right]$$

$$T' = \sqrt{2}\,T$$

So new time period becomes $\sqrt{2}$ times the original time period.

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10.2) A ball is dropped from of certain height on to the floor and keeps bouncing. Is the motion of the ball is S.H.M?

Ans. In this case the acceleration of the ball is not directed towards mean position and its velocity is also not maximum at mean position. So we can say that motion of a bouncing ball is not Simple Harmonic Motion.

10.3) A students performed two experiments with a simple pendulum. He used two bobs of different masses by keeping other parameters constant. To his astonishment the time period of the pendulum did not change. Why?

Ans. We know that Time period of simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

From above relation we see that time period of simple pendulum does not depends upon its mass. Therefore if we use two bobs with different masses then time period does not change.

10.4) What type of waves do not require any material medium for their propagation.

Ans. Electromagnetic waves are such waves which do not requires any medium for their propagation. Light waves, heat waves, radio waves, t.v waves and x-rays are examples of electromagnetic waves.

10.5) Plane wave in the ripple tank undergo refraction when they move from deep to shallow water. What change occurs in the speed of the waves?

Ans. When plane waves enter from deep to shallow water then their wavelength decreases. Due to this decreases in their wavelength their speed also decreased.

NUMERICAL PROBLEMS

10.1) The time period

Data:

Time period $= \mathrm{T} = 2\mathrm{sec}$

Value of ‘g’ on earth = $g_e = 10\, m/s^2$

Value of ‘g’ on moon = $g_m = \frac{g_e}{6} = \frac{10}{6} = 1.6\, m/s^2$

$lenght\ on\ earth = le = ?$

$length\ on\ moon - lm = ?$

Solution:

We know that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For length on earth

$$T = 2\pi\sqrt{\frac{l_e}{g_e}}$$

$$2 = 2(3.14)\sqrt{\frac{l_e}{10}}$$

$$2 = 6.28\sqrt{\frac{l_e}{10}}$$

$$\frac{2}{6.28} = \sqrt{\frac{l_e}{10}}$$

$$0.31847 = \sqrt{\frac{l_e}{10}}$$

For length on moon

$$T = 2\pi\sqrt{\frac{l_m}{g_m}}$$

$$2 = 2(3.14)\sqrt{\frac{l_m}{1.6}}$$

$$2 = 6.28\sqrt{\frac{l_m}{1.6}}$$

$$\frac{2}{6.28} = \sqrt{\frac{l_m}{1.6}}$$

$$0.31847 = \sqrt{\frac{l_m}{1.6}}$$

Squaring b.s

Squaring b.s

$$(0.31847)^2 = \left(\sqrt{\frac{l_e}{10}}\right)^2$$

$$0.1014 = \frac{l_e}{10}$$

$$0.1014\ x\ 10 = l_e$$

$$l_e = 1.014m$$

$$(0.31847)^2 = \left(\sqrt{\frac{l_m}{1.6}}\right)^2$$

$$0.1014 = \frac{l_m}{1.6}$$

$$0.1014\ x\ 1.6 = l_m$$

$$l_m = 0.16224m$$

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10.2)

<u>Data</u>:

$length\ of\ pendulum = \text{l} = 0.99\text{m}$

$Time\ period = T = 4.9\ sec$

Value of 'g' on moon = g_m = ?

Solution:

We know that time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For moon put $g = g_m$

$$T = 2\pi\sqrt{\frac{l}{g_m}}$$

$$4.9 = 2(3.14)\sqrt{\frac{0.99}{g_m}}$$

$$4.9 = 6.28\sqrt{\frac{0.99}{g_m}}$$

$$\frac{4.9}{6.28} = \sqrt{\frac{0.99}{g_m}}$$

$$0.78025 = \sqrt{\frac{0.99}{g_m}}$$

Squaring b.s

$$(0.78025)^2 = \left(\sqrt{\frac{0.99}{g_m}}\right)^2$$

$$0.60879 = \frac{0.99}{g_m}$$

$$g_m = \frac{0.99}{0.60879}$$

$$g_m = 1.626\, m/s^2$$

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10.3)

<u>Data</u>:

Length of pendulum = l = 1m

Value of ‘g’ on earth = $g_e = 10\, m/s^2$

Value of ‘g’ on moon = $g_m = \frac{g_e}{6} = \frac{10}{6} = 1.6\, m/s^2$

$Time\ period\ on\ earth = T_e = ?$

$Time\ period\ on\ moon = T_m = ?$

Solution:

We know that time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For Earth

$$T_e = 2\pi\sqrt{\frac{l}{g_e}}$$

$$T_e = 2(3.14)\sqrt{\frac{1}{10}}$$

$$T_e = 6.28\sqrt{0.1}$$

$$T_e = (6.28)(0.316)$$

$$T_e = 1.9844\ sec$$

For Moon

$$T_m = 2\pi\sqrt{\frac{l}{g_m}}$$

$$T_m = 2(3.14)\sqrt{\frac{1}{1.6}}$$

$$T_m = 6.28\sqrt{0.625}$$

$$T_m = (6.28)(0.79056)$$

$$T_m = 4.96\ sec$$

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10.4)

<u>Data</u>:

$Time\ period = T = 2sec$

$length\ of\ pendulum = l = ?$

Value of 'g' = $g_m = 10\, m/s^2$?

Solution:

We know that time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For moon put $g = g_m$

10.5

<u>Data</u>:

$Number\ of\ waves = 100$

Time = t = 20sec

$wavelength = \lambda = 6cm$

$= \lambda = \frac{6}{100} = 0.06m$

$frequency = F = ?$

$ime\ period = T = ?$

$Wavelength = V = ?$

Solution:

We know

Frequency = <u>Number of waves</u>

$$T = 2\pi\sqrt{\frac{l}{g_m}}$$

$$4.9 = 2(3.14)\sqrt{\frac{0.99}{g_m}}$$

$$4.9 = 6.28\sqrt{\frac{0.99}{g_m}}$$

$$\frac{4.9}{6.28} = \sqrt{\frac{0.99}{g_m}}$$

$$0.78025 = \sqrt{\frac{0.99}{g_m}}$$

Squaring b.s

$$(0.78025)^2$$

$$= \left(\sqrt{\frac{0.99}{g_m}}\right)^2$$

$$0.60879 = \frac{0.99}{g_m}$$

$$g_m = \frac{0.99}{0.60879}$$

$$g_m = 1.626\, m/s^2$$

Time

$$F = \frac{100}{20}$$

$$\boxed{F = 5\text{Hz}}$$

Now for 'T'

$$T = \frac{1}{F}$$

$$T = \frac{1}{5}$$

$$T = 0.2\ sec$$

Now for 'V' using formula

$$V = F\ \lambda$$

$$V = (5)(0.06)$$

$$V = 0.3\ m/s$$

10.6)

Data:

$$Frequency = F = 100$$

$$wavelength = \lambda = 3cm$$

$$= \lambda = \frac{3}{100} = 0.03m$$

$$Wavelength = V = ?$$

Solution:

10.7)

Data:

$$Frequency = F = 190\ Hz$$

$$Length\ of\ spring = Distance = 90$$

$$\text{Time} = \text{t} = 0.5 \text{ sec}$$

$$Time\ period = T = ?$$

$$Wavelength = V = ?$$

$$Wavelength = \lambda = ?$$

We know that

$V = F\lambda$

$V = (12)(0.03)$

$V = 0.36\, m/s$

Solution:

For Time period

We know

$T = \frac{1}{F}$

$T = \frac{1}{190}$

$T = 0.005 \quad OR \quad T = 0.1\, sec$

Now for 'V' we know we know

$Speed = \frac{Distance}{Time}$

$V = \frac{90}{0.5}$

$\text{V} = 180\, m/s$

Now for 'λ'

$V = F\lambda$

$180 = (190)\lambda$

$\frac{180}{190} = \lambda$

$\lambda = 0.947\, m$

10.8)

<u>Data</u>:

$Wavelength = \lambda = 6\, cm$

$\lambda = \frac{6}{100} = 0.06\, m$

$Frequency = F = 4.8\, m$

$\text{Speed} = \text{V} = ?$

$Time\ period = T = ?$

Solution:

We know

$T = \frac{1}{F}$

$T = \frac{1}{4.8}$

$T = 0.21\, sec$

Now for 'V' we know

$V = F\lambda$

10.9)

<u>Data</u>:

$Distance = S = 80\, cm$

$S = \frac{80}{100} = 0.8\, m$

$Frequency = F = 5\, Hz$

$Wavelength = \lambda = 40\, mm$

$\lambda = \frac{40}{1000} = 0.04\, m$

$Time\ period = T = ?$

To find time first we will find speed by using formula

$V = F\lambda$

$V = (5)(0.04)$

$V = 0.2\, m/s$

Now we find time

$V = (4.8)(0.06)$
$V = 0.288\ m/s$

$Speed = \frac{Distance}{Time}$
$0.2 = \frac{0.8}{t}$
$t = \frac{0.8}{0.2}$
$T = 4\ sec$

10.10)

<u>Data</u>:

$Frequency = F = 90\ MH$
$F = 90\ x\ 10^6\ Hz$
$Speed = V = 3\ x\ 10^8\ m/s$
$Wavelength = \lambda = ?$

We know

$V = F\ \lambda$
$3\ x\ 10^8 = (90\ x\ 10^6)\lambda$
$\frac{3\ x\ 10^8}{90\ x\ 10^6} = \lambda$
$\frac{3}{90}10^{8-6} = \lambda$
$0.0333\ x\ 10^2 = \lambda$
$\lambda = \ 0.0333\ x\ 10^2$
$\lambda = 3.33\ m$